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Determination of the Permissible Solutions Area by Image Reconstruction from a few Projections: Method 2-CLEAN DSA

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Abstract. We have proposed the 2-CLEAN DSA (Determination of Solution Area) method for the estimation of the area of possible images (from the "obtuse" (smooth) to the "sharp" variants) in complicated cases with constraints and poor *a priori* information. The area of permissible solutions can be determined with the help of two CLEAN algorithms: standard CLEAN and Trim Contour CLEAN (TC-CLEAN). The procedure has high efficiency and simple criteria by errors minimization of initial and control 1-D profiles. We present here a description of some valuable features of the reconstruction technique.

1. Introduction

Iterative algorithms with non-linear constraints are very attractive in image reconstruction with only a few strip-integrated projections (Vasilenko & Taratorin 1986). The process of convergence to solutions for the various realizations of iterative schemes using the different versions of the CLEAN algorithm have already been investigated for this problem (Agafonov & Podvojskaya 1989). Two dimensional image reconstruction from 1-D projections is often hampered by the small number of available projections, by an irregular distribution of position angles, and by positions angles that span a range smaller than about 80 degrees. These limitation are typical of both lunar occultations of celestial sources and observations with the fan beam of radio telescope, and also apply to greatly foreshortened reconstructive tomography. Our previous paper (Agafonov 1997) also contains the basic description of the problem and the features of 2-CLEAN DSA method. This paper contains some examples and useful diagrams and also a valuable addition for the development of the reconstruction technique for this problem.

2. Application

The problem requires the solution of the equation

$$G = H * F \ (+noise) \ , \qquad (1)$$

where $F(x,y)$ is the object brightness distribution, $H(x,y)$ is the fan (dirty) beam, and $G(x,y)$ is the dirty (summary) image. An example of the fan beam

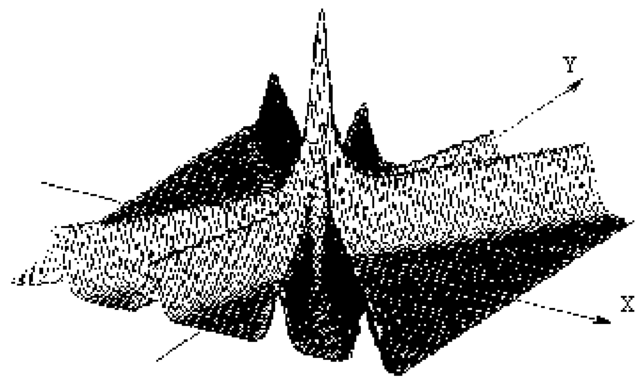

Figure 1. Fan beam $H(x, y)$ for four projections (typical example).

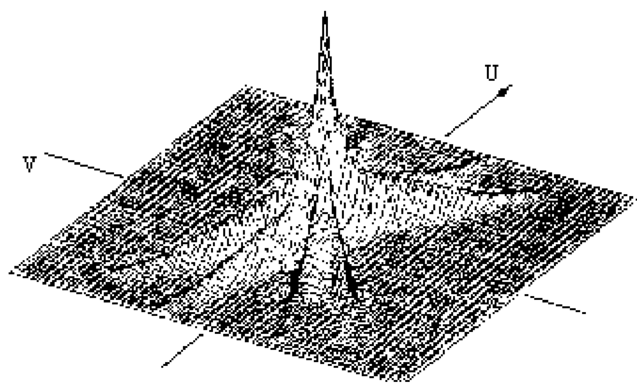

Figure 2. The same fan beam $H(u, v)$ on the UV plane (sampling of about 0.1 of the UV plane in the limit ω_l to the fixed radius of the frequency).

for four projections is shown in Figure 1. The classical case (Bracewell & Riddle 1967) needs a number of projections $N \geq \pi D/\varphi$, where φ is the desired angular resolution, and D is the diameter of the object. The incomplete sampling of $H(u, v)$ (see Figure 2) requires the extrapolation of the solution using non-linear processing methods.

A two dimensional image reconstruction of a complicated object using a poor fan beam function $H(u, v)$ needs to carry out the following procedure:

1. **Test experiment**: evaluation of the possibility and quality of restoration on the similar images versions; modeling for available projections (number N, position angles, signal/noise ratio) and desirable frequency limit ω_l. This point can include following steps:
   - **2-D object model** $\rightarrow$ **1-D profiles** $\rightarrow$ **Dirty image**
   - **CLEAN** (λ) or **TC-CLEAN** (λ, TC) using of **Fan beam**

- **Control test** from clean maps: **Calculation of** σ (ERROR of control and initial 1-D profiles)
- **Correction of** λ **or** λ, TC **to** $min\ \sigma$**.** This step can help to determine the best algorithm parameters range.

The process of solutions convergence using CLEAN (Hogbom 1974) and Trim Contour CLEAN (TC-CLEAN) (Steer et al. 1984) has been analyzed (Agafonov & Podvojskaya 1989; Agafonov & Podvojskaya 1990) by optimizing the parameter λ or λ, TC (Trim Contour level) to $min\ \sigma$. A real example of such a process for both algorithms is shown graphically in Figures 3 and 4.

2. **Reconstruction from real observational 1-D profiles**. This process can also include the correction of λ or λ, TC (Trim Contour level) to $min\ \sigma$.

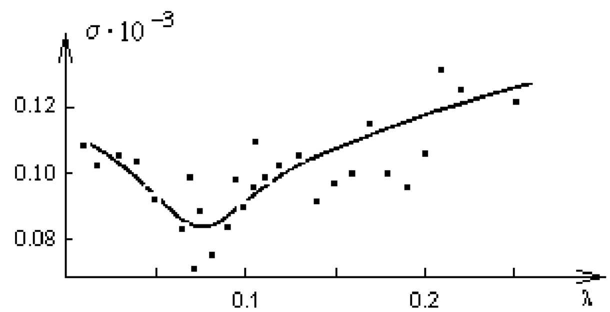

Figure 3. The error of original and control profiles as a function of the loop gain (using standard CLEAN).

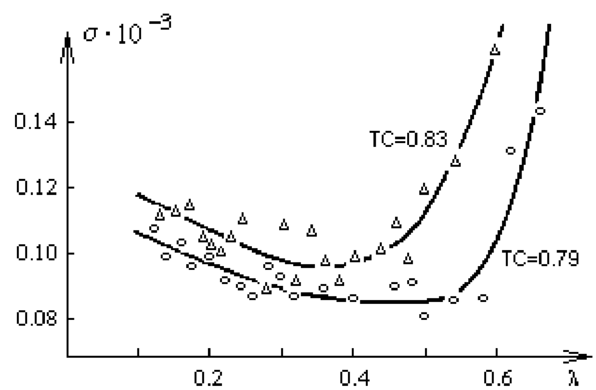

Figure 4. The error of original and control profiles as function of the loop gain loop using two different Trim Contour levels of TC-CLEAN.

3. Conclusions

CLEAN forms the solution from the sum of peaks and the result is the sharpest variant permissible within the established constraints. On the other hand TC-CLEAN accumulates its result from the most extended components that satisfy the constraints, producing the smoothest solution (Agafonov & Podvojskaya 1990).

A simple object (consisting of the peaks) may be successfully restored by the standard CLEAN. The results obtained by both methods are practically identical for a simple object consisting of individual components, but TC-CLEAN is more computationally efficient. For smoothed 1-D profiles with small "hillocks", the solution can be obtained from the isolated individual components (CLEAN), and also from the more smoothed components (TC-CLEAN). CLEAN increases the contrast of small components, but the extended background decreases because of "grooves". **If** $min\ \sigma$ **(CLEAN)** $\cong$ $min\ \sigma$ **(TC-CLEAN)** (see the example shown in Figures 3 and 4), **the solutions will be formally equivalent** for both algorithms, and so we have two choices: (*i*) to prefer the result corresponding to the physical peculiarities of the object in accordance with *a priori* information; or (*ii*) to assume the existence a probable class of solutions between the "smooth" one from TC-CLEAN and the "sharp" one from CLEAN.

The area of permissible solutions of complicated objects can be determined with a help of both algorithms. The 2-CLEAN DSA procedure can show a range of possible images from "smooth" to "sharp" variants satisfying imposed constraints and poor *a priori* information.

The application of TC-CLEAN and CLEAN was presented as a reconstruction of the Crab Nebula map at 750 MHz (Agafonov et al. 1990). The method of 2-CLEAN DSA allowed us to determine that the area of the permissible solutions lies formally between the "sharp" (CLEAN) and "smooth" (TC-CLEAN) variants. Two maps were generally similar. The standard CLEAN increased the contrast of small components while the TC-CLEAN map gave a better agreement with known *a priori* information.

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References

Agafonov, M. I. 1997, in ASP Conf. Ser., Vol. 125, Astronomical Data Analysis Software and Systems VI, ed. Gareth Hunt & H. E. Payne (San Francisco: ASP), 202

Agafonov, M. I., & Podvojskaya, O. A. 1989, Izvestiya VUZ. Radiofizika, 32, 742

Agafonov, M. I., & Podvojskaya, O. A. 1990, Izvestiya VUZ. Radiofizika, 33, 1185

Agafonov, M. I., Ivanov, V. P., & Podvojskaya, O. A. 1990, AZh, 67, 549

Bracewell, R. N., & Riddle, A. C. 1967, ApJ, 150, 427

Hogbom, J. A. 1974, A&AS, 15, 417

Steer, D. G., Dewdney, P. E., & Ito, M. R. 1984, A&A, 137, 159

Vasilenko, G. I., & Taratorin A. M. 1986, Image Restoration (in Russian), Radio i svyaz', Moscow.