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Modelling Spectro-photometric Characteristics of Nonradially Pulsating Stars

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Abstract.
We present a computing code for modelling energy flux distributions, photometric indices and spectral line profiles of (non-)radially pulsating Main-Sequence stars. The model is based on the perturbation-expansion formalism taking into account geometrical and nonadiabatic effects.

1. Introduction

For a given mode of oscillation the harmonic time dependence, $\exp(\mathrm{i}\omega_{nlm}t)$, and spherical harmonic horizontal dependence, $Y_l^m(\theta,\phi)$, are assumed for the first order perturbed quantities. The mass displacement for the spheroidal modes is described by y- and z-eigenfunctions and for toroidal modes by τ-eigenfunctions, cf. Dziembowski & Goode (1992). In the case of slowly rotating stars one can use the zero-rotation approximation to describe stellar pulsations. Such a model was used already by Cugier, Dziembowski & Pamyatnykh (1994) to study nonadiabatic observables of β Cephei stars. Apart from $y_{nlm}(r)$ and $z_{nlm}(r)$ it is desirable to use the eigenfunction $p_{nlm}(r)$, connected with the Lagrangian perturbation of pressure, and the $f_{nlm}(r)-$eigenfunction, which describes the variations of the local luminosity. In the nonadiabatic theory of pulsation the eigenvalues ω_{nlm} and the eigenfunctions are complex (cf. e.g., Dziembowski 1977) and $\psi_{nlm} = \arg(f_{nlm}/y_{nlm})$ is the phase lag between the light and radius variations.

2. Continuum Flux Behaviour

2.1. Numerical Integration

The monochromatic flux of radiation is given by

$$\mathcal{F}_\lambda = \int I_\lambda(r,\theta,\phi,\vec{o}\cdot\vec{n})\vec{o}\cdot\vec{n}\frac{dS}{R^2}. \tag{1}$$

where $\vec{o}\cdot\vec{n}$ is the scalar product of the observer's direction, $\vec{o}$, and the normal vector, $\vec{n}$, and dS- the area of the surface element.

In the program the specific intensity data for the new generation line-blanketed model atmospheres of Kurucz (1996) were used in order to study the continuum flux behaviour and photometric indices. Kurucz's (1994) data contain monochromatic fluxes for 1221 wavelengths and monochromatic intensities at 17 points of $\tilde{\mu} = \vec{o} \cdot \vec{n}$. Using these data one can interpolate the monochromatic intensities for the local values of $T_{\rm eff}$, $\log g$ and $\tilde{\mu}$. We can also introduce the linear or quadratic shape for the limb-darkening law as defined by Wade & Rucinski (1985).

2.2. Semi-analytical Method

Integrating Eq.1 over the surface in the linear approach we can obtain the semi-analytical solution, cf. Daszynska & Cugier (1997) for details,

$$\frac{\Delta\mathcal{F}_\lambda}{\mathcal{F}_\lambda^0} = \varepsilon d_{lm0} N_l^0 \Big[(T_1+T_2)\cos((\omega_{nlm}-m\Omega)t+\tilde{\psi}_{nlm}) + (T_3+T_4+T_5)\cos(\omega_{nlm}-m\Omega)t \Big]. \tag{2}$$

In this formula the temperature effects are described by two terms T_1 and T_2, whereas the effects of the pressure changes during the pulsation cycle are included in T_4 and T_5. The T_2 and T_5 terms reflect the sensitivity of the limb-darkening parameters to temperature and gravity variations, respectively. T_3 corresponds to the geometrical effects. N_l^0 is a normalizing factor.

3. Accuracy of the Model Calculations

We examined how the results are influenced by different methods of integration over the stellar surface. The following cases were considered:

- Model 1: the semi-analytical method (Eq.2) with the quadratic form for the limb-darkening law,
- Model 2: the numerical integration of Eq.1 with the quadratic form for the limb-darkening law; constant limb-darkening coefficients corresponding to the equilibrium model were assumed,
- Model 3: the same as Model 2, but the limb-darkening coefficients were interpolated for local values of $T_{\rm eff}$ and $\log g$,
- Model 4: numerical integration over stellar surface with specific intensities interpolated for the local values of $T_{\rm eff}$, $\log g$ and $\tilde{\mu}$.

As an example we consider the energy flux distribution and nonadiabatic observables for a β Cep model. We chose the stellar model ($\log T_{\rm eff}^0 = 4.33668$, $\log g^0 = 4.07842$) calculated with OPAL opacities. This model shows unstable $l = 0$, 1 and 2 modes of oscillations. We calculated theoretical fluxes and the corresponding Strömgren photometric indices at pulsating phases $\varphi = 0.05\ n$ (n=0,...,20). Subsequently amplitudes and phases of the light curves were computed by the least-square method. The accuracy of these calculations can be estimated from Table 1, which gives the results for the Models 1 - 4. The calculations were made on Sun Ultra 1 (192 MB RAM, 166 MHz) computer. The CPU time per 1 pulsating stellar model is from about 2 seconds (for Model 1) to about 10 hours (for Model 4).

Table 1. Nonadiabatic observables.

	l	$A_y{}^*$	φ_y	A_u/A_y	$\varphi_u - \varphi_y$	$\frac{A_{u-y}}{A_y}$	$\varphi_{u-y} - \varphi_y$
Model 1	0	0.0211	3.3166	2.0024	-0.0381	0.8241	-0.0701
Model 2	0	0.0211	3.3167	2.0000	-0.0381	0.8220	-0.0715
Model 3	0	0.0213	3.3167	2.0000	-0.0381	0.8220	-0.0715
Model 4	0	0.0211	3.3168	2.0000	-0.0381	0.8217	-0.0718
Model 1	1	0.0207	3.1916	1.5958	0.0004	0.4876	0.0038
Model 2	1	0.0268	3.1929	1.6119	0.0002	0.4975	0.0022
Model 3	1	0.0268	3.1929	1.6112	0.0002	0.4975	0.0022
Model 4	1	0.0222	3.1910	1.5526	0.0009	0.4535	0.0048
Model 1	2	0.0204	3.2077	1.3476	0.0164	0.2560	0.0790
Model 2	2	0.0195	3.2083	1.3457	0.0161	0.2568	0.0801
Model 3	2	0.0195	3.2083	1.3457	0.0161	0.2568	0.0801
Model 4	2	0.0077	3.2084	1.3170	0.0160	0.2459	0.0804

*assumed

4. Line Profiles

The velocity field of pulsating stars may be found by calculating the time derivative of the Lagrangian displacement. Including the first order effect, the radial component v_p as seen by a distant observer is:

$$v_p = \vec{v}_{puls} \cdot (-\mathbf{e}_z) = \mathrm{Re}\{\mathrm{i}\omega_{nlm}[\cos\theta\delta r(R,\theta,\phi,t) - r\sin\theta\delta\theta(R,\theta,\phi,t)]\}$$

$$= \omega_{nlm}\Big[\cos\theta r[y_{nlm}(r)+\frac{2m\Omega}{\omega^0_{nlm}}\tilde{y}_{nlm}(r)]\sum_{k=-l}^{l} d_{lmk}(i)N_l^k P_l^k(\theta)\sin((\omega_{nlm}-m\Omega)t+k\phi)$$

$$-r\sin\theta\Big([z_{nlm}(r)+\frac{2m\Omega}{\omega^0_{nlm}}\tilde{z}_{nlm}]\sum_{k=-l}^{l} d_{lmk}(i)N_l^k\frac{\partial P_l^k(\theta)}{\partial\theta}\sin((\omega_{nlm}-m\Omega)t+k\phi)$$

$$+\frac{\tau'{}_{l+1,m}}{\sin\theta}\sum_{k=-(l+1)}^{l+1} d_{l+1,m,k}(i)kN_{l+1}^k P_{l+1}^k(\theta,\phi)\cos((\omega_{nlm}-m\Omega)t+k\phi)$$

$$+\frac{\tau'{}_{l-1,m}}{\sin\theta}\sum_{k=-(l-1)}^{l-1} d_{l-1,m,k}(i)kN_{l-1}^k P_{l-1}^k(\theta,\phi)\cos((\omega_{nlm}-m\Omega)t+k\phi)\Big)\Big]. \quad (3)$$

The radial velocity due to pulsation and rotation is then

$$v_{\mathrm{r}} = v_{\mathrm{p}} - v_{\mathrm{e}}\sin i\,\sin\theta\,\sin\phi, \quad (4)$$

where v_{e} corresponds to the equatorial velocity of rotation and i is the angle between the rotation axis and the direction to the observer.

We illustrate the predicted behaviour of Si III 455.262 nm line profiles for stellar model given in Sect.3. We considered Kurucz's (1994) model atmospheres

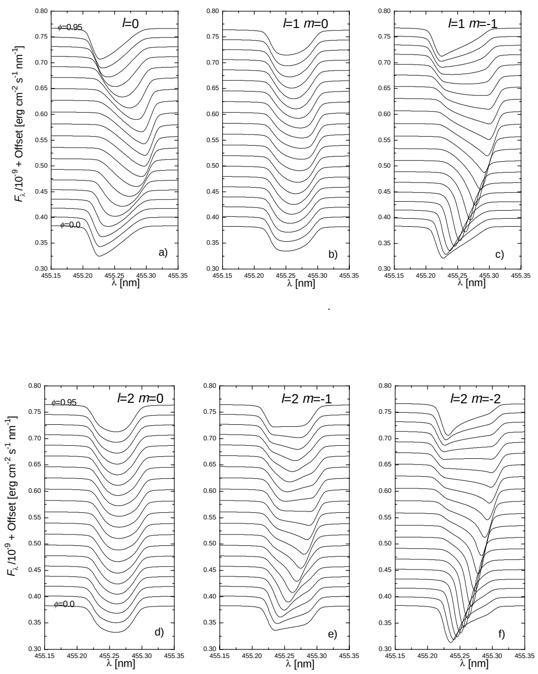

Figure 1. Theoretical SiIII 455.262 nm line profiles for various modes.

with the solar chemical composition and the microturbulent velocity $v_{\rm t} = 0$. All calculations were made for the amplitude of the stellar radius variations $\varepsilon = 0.01$ and rigid rotation. Figures $1a - f$ show the theoretical line profiles for different phases of pulsation for $i = 77^\circ$ and the equatorial velocity $V_{\rm e} = 25$ km s^{-1}. The spectra are given in absolute units. In order to avoid overlap, vertical offsets were added to each spectrum using the relationship: $\mathcal{F}_\lambda + n \cdot 0.02 \cdot 10^{-9}$.

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