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Methodology of Time Delay Change Determination for Uneven Data Sets

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Abstract. At the previous ADASS Conference (Oknyanskij, 1997a) we considered and used a new algorithm for time-delay investigations in the case when the time delay was a linear function of time and the echo response intensity was a power-law function of the time delay. We applied this method to investigate optical-to-radio delay in the double quasar 0957+561 (generally accepted to be a case of gravitational lensing). It was found in this way that the radio variations (5 MHz) followed the optical ones, but that the time delay was a linear function of time with the mean value being about 2370 days and with the rate of increase $\mathbf{V} \approx 110$ days/year.

Here we use Monte-Carlo simulations to estimate the significance of the results. We estimate (with the 95% confidence level) that the probability of getting the same result by chance (if the real $\mathbf{V}$ value was equal to 0 days/year) is less then 5%. We also show that the method can also determine the actual rate of increase $\mathbf{V_a}$ of the time delay in artificial light curves, which have the same data spacing, power spectrum and noise level as real ones.

We briefly consider some other possible fields for using the method.

1. Introduction

At the previous ADASS Conference (Oknyanskij, 1997a, see also Oknyanskij 1997b) we considered and used a new algorithm for time-delay investigations in the case when the time delay was a linear function of time and the echo response intensity was a power-law function of the time delay. We applied this method to investigate optical-to-radio delay in the double quasar 0957+561 (generally accepted to be a case of gravitational lensing). It was found in this way that the radio variations (5 MHz) followed the optical ones, but that the time delay was a linear function of time with the mean value being about 2370 days and with the rate of increase $\mathbf{V} \approx 110$ days/year.

The cross-correlation function for this best fit is shown in Figure 1 together with the cross-correlation function for the data without any corrections for possible variability of the time delay value. The maximum of the cross-correlation for the last case (if $\mathbf{V} = 0$ days/year) is less then 0.5. So we can note that our fit explains the real data significantly better then the simple model with some constant time delay. Meanwhile Monte-Carlo simulations are needed to estimate

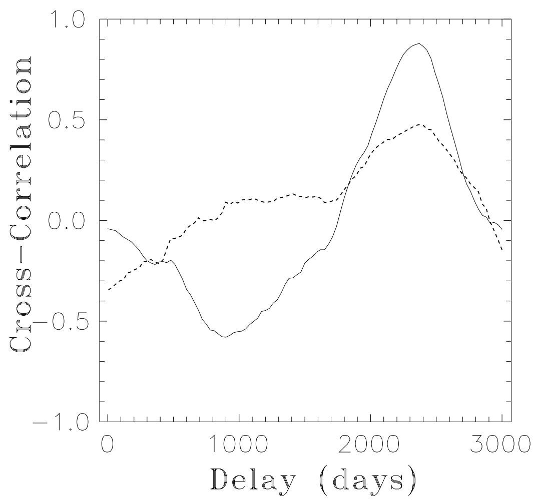

Figure 1. Cross-correlation functions for combined radio and optical light curves. The solid curve shows the cross-correlation function with correction of the data for the best fits parameters: $\mathbf{V} = 110$ days/year and $\alpha = 0.7$ (Oknyanskij, 1997a,b). The dashed curve shows the simple cross-correlation function for the data without any correction i.e., taking $\mathbf{V}$=0 days/year.

the significance of our result. The methodology of these estimations is briefly explained below.

2. Monte-Carlo Simulations

The Monte-Carlo method is a powerful tool that can be used to obtain a distribution-independent test in any hypothesis-testing situation. It is particularly useful for our task, because it is very difficult or practically impossible to use some parametric way to test some null against the alternative hypothesis. We used the Monte-Carlo method to estimate the significance of the result about optical-to-radio delay (Oknyanskij & Beskin, 1993) obtained on the basis of observational data in the years 1979-1990 . With 99% confidence level we rejected the possibility that the high correlation could be obtained if the optical and radio variations were really independent. In a subsequent paper we will estimate the probability of obtaining our result by chance (Oknyanskij 1997a,b) applying the new method to observational data obtained in the years 1979-1994. Here we assume that the optical and radio data are correlated and estimate the significance of the result about possible variability of the time delay value. So here, under H_0, optical and radio data are correlated but there is some constant time delay and the absolute maximum of the correlation coefficient obtained for some $\mathbf{V} \geq \mathbf{V_t} > 0$ days/year reflects random variation. We take $\mathbf{V_t} = 110$ days/year as the test value. When H_1 is true, the time delay is not some constant, but some increasing function of time. We need to calculate the distribution of $\mathbf{V}$ under the null hypothesis. The $p(\mathbf{V})$ value is probability, under H_0, of obtaining some $\mathbf{V}'$ value at least as extreme as (i.e., bigger or equal to) the $\mathbf{V}$. The smaller the $p(\mathbf{V})$ value the more likely we are to reject H_0 and accept H_1. If $p(\mathbf{V_t})$ is less than the typical value of 0.05 then H_0 can be rejected.

The idea of our Monte-Carlo test is the following:

1. We produce m=500 pairs of the simulated light curves which have the same power spectra, time spacing, signal/noise ratio as the real optical and radio data, but with constant value of time delay (τ_{or} = 2370 days) and about the same maximum values of cross-correlation functions.

2. We apply the same method and make all the steps the same as was done for the real data (Oknyanskij, 1997a,b), but for each of m pairs of the simulated light curves. The proportion $\mathbf{p(V)}$ of obtained $\mathbf{V}'$ (see Figure 2) that yield a value bigger or equal to $\mathbf{V}$ provides an estimate of the $p(\mathbf{V})$ value. When $m \geq 100$, standard error of the estimated $p(\mathbf{V})$ value can be approximated by well-known formula $\sigma = [\mathbf{p(1-p)}/m]^{1/2}$ (see Robbins and Van Ryzin 1975). An approximate 95% confidence interval for the true p value can be written as $\mathbf{p} \pm 2\sigma$. As it is seen from Figure 2 - $\mathbf{p(V_t)} \approx 3\%$. The approximate standard error of this value is about 0.8%. We can write the 95% confidence interval for $p(\mathbf{V_t}) = (3 \pm 1.6)\%$ and conclude with a 95% confidence level that $p(\mathbf{V_t}) \leq 5\%$. So the H_0 can be rejected, i.e., time delay is some increasing function of time. For the first step we assume that it is a linear function of time and found $\mathbf{V}$ = 110 days/year, a value approximately equal to the true one.

3. To show that the method has real abilities to determinate a value of of the time delay rate of increase in the light curves we again use Monte-Carlo simulation as it is explained in (1) and (2), but the actual $\mathbf{V_a}$ value is 110 days/year. Then we obtain the histogram (Figure 3), which shows the distribution of obtained $\mathbf{V}'$ values. It is clear that the distribution has some asymmetry, which could be a reason for some small overestimation of $\mathbf{V}$ value, since the mathematical expectation of mean $\mathbf{V}$' is about 114 days/year. Meanwhile we should note that the obtained histogram shows us the ability of the method to get the approximate estimate of the actual $\mathbf{V_a}$ value, since the distribution in the Figure 3 is quite narrow. Using this histogram we approximately estimate the standard error $\sigma(\mathbf{V}) \approx 15$ days/year (for $\mathbf{V}$ value, which has been found for Q 0956+561).

3. Conclusion and Ideas for Possible Applications of the Method

We have found a time delay between the radio and optical flux variations of Q 0957+561 using a new method (Oknyanskij, 1997a,b), which also allowed us to investigate the possibilities that (1) there is some change of time delay that is a linear function of time, and (2) the radio response function has power-law dependence on the time delay value. Here we estimate the statistical significance of the result and the ability of the method to find the actual value of $\mathbf{V}$ as well as its accuracy. We show that with 95% confidence level the probability of getting a value of $\mathbf{V} \geq 110$ days/year (if actual $\mathbf{V_a}$ would be equal to 0 days/year) is less then 5%. We estimate that standard error of the $\mathbf{V}$ value (which has been found for Q 0957+561) is about 15 days/year.

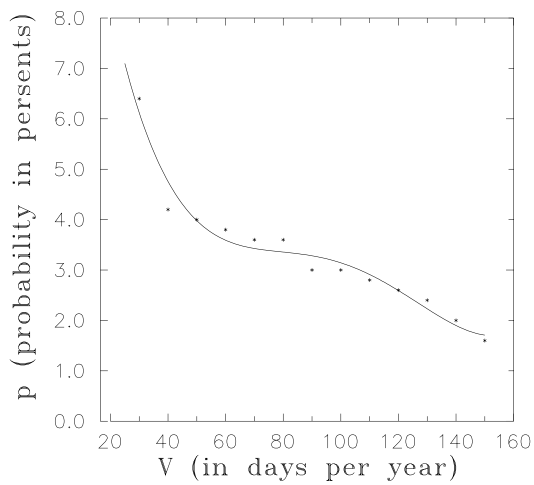

Figure 2. $\mathbf{p(V)}$ is probability to get some value $\mathbf{V'} \geq \mathbf{V}$ if the actual $\mathbf{V_a} = 0$ days/year. All Monte-Carlo test $\mathbf{p}$ values are based on applying our method to 500 pairs of artificial light curves with the actual parameters $\tau_{or}(t_0) = 2370$ days and $\mathbf{V_a} = 0$ days/year.

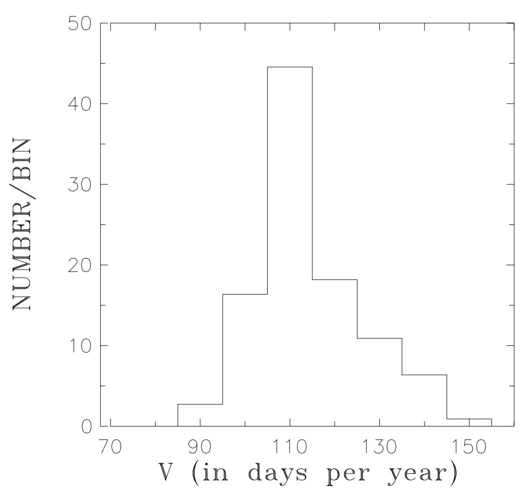

Figure 3. Result of Monte-Carlo test (see Figure 2) with actual values of $V_a = 110$ days/year and $\tau_{or}(t_0) = 2370$ days. The histogram shows distribution of obtained $\mathbf{V'}$ values.

Finally, we can briefly note some other fields where the method may be used:

1. Time delay between continuum and line variability in AGNs may be a function of time as well as the response function possibly being some function of time. So our method can be useful for this case.

2. Recently, it has been suggested by Fernandes et. al (1997) that variability of different AGNs might have coincident recurrent patterns in their light curves. However it has been found that the time-scales for these patterns in NGC 4151 and 5548 are about the same, there are a lot of reasons to expect that the patterns in AGN light curves may be similar, but have different time scales. It is possible to use our method with some enhancements to investigate this possibility. Some Monte-Carlo estimations of the significance would be also very useful. The probability that these common recurrent patterns in AGNs occur by chance should be estimated.

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