Astronomical Data Analysis Software and Systems VII
ASP Conference Series, Vol. 145, 1998
R. Albrecht, R. N. Hook and H. A. Bushouse, eds.

Substepping and its Application to HST Imaging

Nailong Wu and John Caldwell

Dept. of Physics and Astronomy, York University, 4700 Keele St., North York, Ontario, M3J 1P3, Canada

Abstract. The substepping technique is used for the Hubble Space Telescope (HST) imaging to cope with the problem of undersampling and to improve resolution. In this paper, this technique is first introduced in the language of signal/image processing. Then its application to HST: FOS ACQ imaging and WFPC2 subpixel dithering, is described. Its possible use for NICMOS is also discussed.

1. Introduction

The substepping technique is used to ameliorate the problem of undersampling in data acquisition for an imaging system. For the HST, undersampling means that the size of the pixel in a camera is larger than the critical value determined by the optics of the telescope, and aliasing takes place.

Obviously, the problem of undersampling could be resolved if the pixel size could be reduced. However, in many circumstances, the pixel size is fixed. Then, substepping is the only solution to the problem. In essence, substepping means that data are acquired in steps smaller than the pixel size, and then processed to achieve resolution comparable to the step size.

2. Substepping in Data Acquisition and Reconstruction

In this section one-dimensional (1-D) notation and diagrams are used for clarity. Results can be easily extended to the two-dimensional (2-D) case by changing dimensional subscripts and operations to 2-D.

2.1. Image Formation and Sampling

The brightness distribution, $a_0(x)$, in the field of view is convolved with the point spread function (PSF) of the optics of the telescope to form a continuous image, $a(x)$, which has lower resolution than $a_0(x)$ due to the convolution. Resolution can by improved by deconvolution of $a(x)$ with respect to the PSF to restore $a_0(x)$.

Sampling is an operation to integrate $a(x)$ within a pixel, and to assign the result to this pixel (Figure 1A). Each pixel has its nominal position at the center of the integral interval.

2.2. Substepping

To avoid aliasing, the sampling interval Δx must be equal to (critical sampling) or smaller than (oversampling) its critical value Δx_c, which is determined by the highest frequency component in $a(x)$. If $\Delta x > \Delta x_c$ (undersampling), aliasing will take place.

In order to overcome the problem of undersampling (Figure 1A), we may reduce the pixel size by a factor of N, such that $1/N\Delta x \leq \Delta x_c$ (proper sampling, including critical sampling and oversampling; Figure 1D for $N = 2$). We call this *normal sampling* because both the integral and sampling intervals are equal to the pixel size.

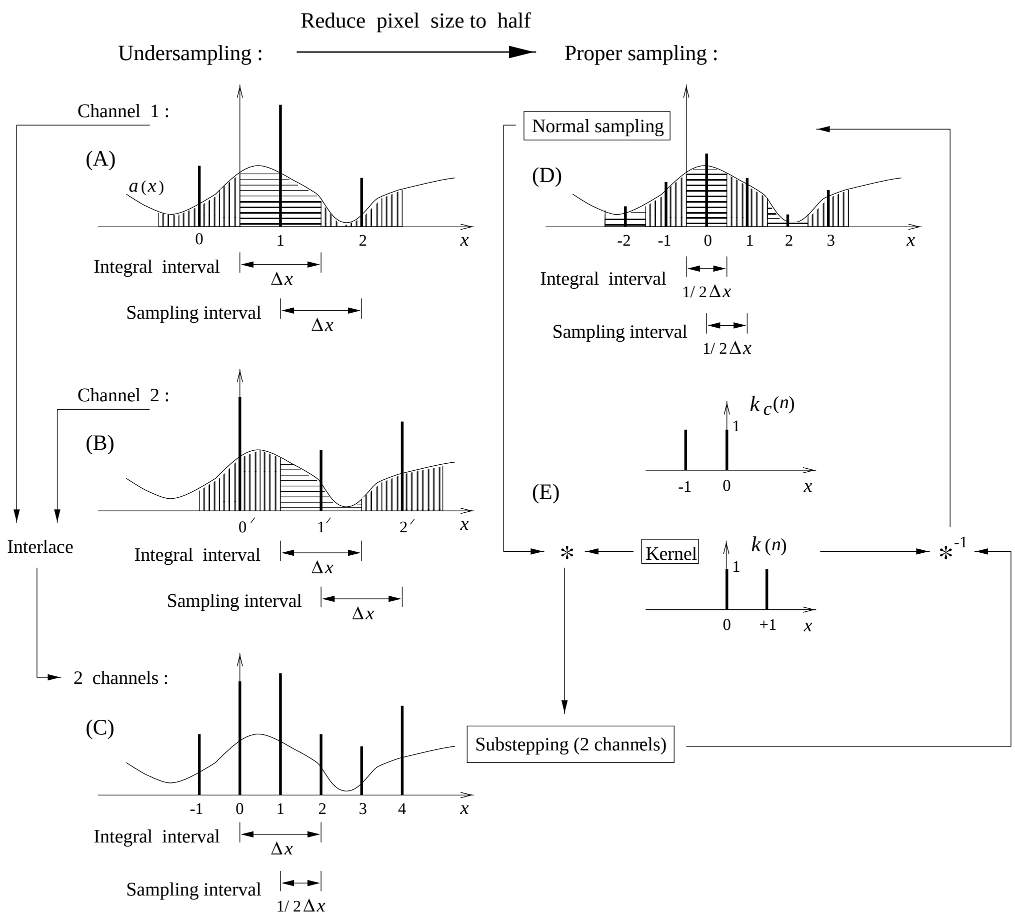

Figure 1. Substepping (N=2) and normal sampling. The vertical bar at a pixel's center represents the integral value. $*$ and $*^{-1}$ denote convolution and deconvolution, respectively.

In the case where reducing the pixel size is impossible, we can use N integral-sampling devices (channels) to acquire data. The resulting N sequences shifted successively by $1/N\Delta x$ (Figure 1A,B) are combined in a manner of interlacing (C), so that the sampling interval becomes $1/N\Delta x$. We call this *subsampling* or *substepping* because the sampling interval is equal to a subpixel size while the integral interval is still equal to the pixel size. If N is sufficiently large such that $1/N\Delta x \leq \Delta x_c$, aliasing will be eliminated, and consequently resolution will be improved.

2.3. Reconstruction Using the Substepped Sequence

Let us compare substepping (Figure 1C) with normal sampling (D), both with the sampling interval $1/N\Delta x$. For the former, the integral interval is larger, and the smoothing effect of integration is stronger. Therefore, its resolution is lower than that of normal sampling.

The substepped sequence is a moving-sum of the normal-sampled sequence. For instance, the pixel value at $x = 1$ in Figure 1C comes from the pixel value at $x = 1$ in A, which is equal to the sum of the pixel values at $x = 0, 1$ in D.

The moving-sum operation is, in fact, a cross-correlation of the normal-sampled sequence with the kernel $k_c(n)$ (Figure 1E). However, this cross-correlation is equivalent to a convolution with the kernel $k(n)$, $k(n) = k_c(-n)$. Therefore, deconvolution of the substepped sequence with respect to the kernel $k(n)$ can be carried out to reconstruct the normal-sampled sequence. As a result, the smoothing effect due to the moving-sum is eliminated and resolution is improved.

In summary, when the pixel size is too large, substepping in data acquisition can be employed to eliminate aliasing. The resulting N sequences from N channels are interlaced. This operation alone can improve resolution. However, the improvement in resolution is made mostly by deconvolution of the substepped sequence with respect to the kernel $k(n)$ to eliminate the moving-sum effect in substepping, i.e., to weaken the smoothing effect of the integration operation.

Deconvolution with respect to the PSF (Sect. 2.1.), which can be used to improve resolution (restore $a_0(x)$ from $a(x)$), is independent of reconstruction of substepped data.

3. Application to HST Imaging

3.1. WFPC2 Subpixel Dithering

Undersampling with WFPC2 occurs because of the large pixel size of the CCD chips. The substepping technique used to overcome this problem is named "subpixel dithering" or simply "dithering" (Figure 2a).

During an observation, the pointing of the telescope is changed so that successive images are shifted along each axis by subpixel amounts. Then, these images are combined to obtain a single image having a smaller pixel size on a finer grid, using the POCS-based method (Adorf 1995), "Drizzling" method (Hook & Fruchter 1997), and deconvolution-based methods like **acoadd**, **crcoad**, **mem**, and **lucy** in IRAF/STSDAS.

3.2. FOS ACQ Imaging

The undersampling problem with FOS arises from the large size of the diodes used for data acquisition. In substepping (Figure 2b), the step of the diode's motion is one quarter of its width in the X-direction, and one sixteenth of its height in the Y-direction.

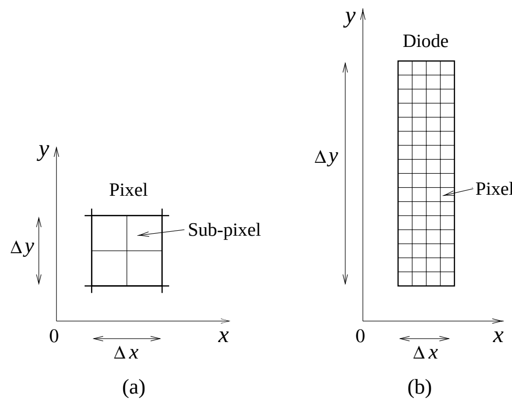

Figure 2. Substepping for HST imaging. (a) WFPC2 dithering ($N_x = N_y = 2$). (b) FOS ACQ imaging ($N_x = 4, N_y = 16$).

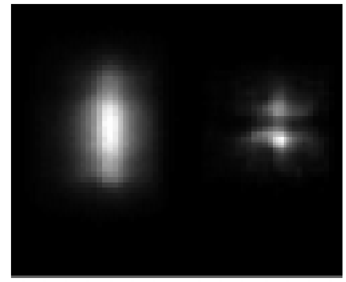

Figure 3. An FOS ACQ image (left) and its reconstruction by **mem** (right).

The diode and the 64 ($= 4 \times 16$) small areas on it would normally be called the pixel and subpixels, respectively, according to standard substepping terminology. However, in this particular case the small areas have commonly, but inaccurately, become known as pixels.

The deconvolution with respect to $k(n_x, n_y)$ is called *reconstruction*. The deconvolution task **mem** or **lucy**, or the direct inversion task **tarestore**, in IRAF/STSDAS can be used for this purpose. **tarestore** results in high level sidelobes (rings) and noise in reconstructed images; **mem** and **lucy** give much better results (Wu & Caldwell 1997).

The smoothing effect due to the large diode size badly reduces resolution in FOS ACQ images. Reconstruction can remove this effect and dramatically improve resolution. Figure 3 shows an image, before and after reconstruction, of a star behind the bar in an FOS barred aperture.

3.3. NICMOS Imaging

In NICMOS imaging, PSFs are critically sampled at $\lambda_c = 1.0$ and 1.75 μm for Cameras 1 and 2, respectively. When a working wavelength is shorter than λ_c, the problem of undersampling occurs. The situation here is similar to WFPC2 in principle. Therefore, the substepping technique can be used.

References

Adorf, H.-M. 1995, in Astronomical Data Analysis Software and Systems IV, A.S.P. Conf. Ser., Vol. 77, eds. R. A. Shaw, H. E. Payne & J. J. E. Hayes (San Francisco: ASP), 456

Hook, R. N. & Fruchter, A. S. 1997, in Astronomical Data Analysis Software and Systems VI, A.S.P. Conf. Ser., Vol. 125, eds. G. Hunt & H. E. Payne (San Francisco: ASP), 147

Wu, N. & Caldwell, J. 1997, in Proceedings of 1997 HST Calibration Workshop, Space Telescope Science Institute, ed. S. Casertano, et al. (in press)