1) Assumed Object Counts

a) High Latitude Case

The high latitude star and K-band galaxy counts used by Stefano seem to be in near perfect agreement with the curves I have assembled from the data available in the literature. For reference, Figure 1 gives my estimate for the total integrated source counts toward the galactic poles.

The galaxy counts in this figure were assembled by numerically integrating an eyeball fit to the synopsis of the available differential K-band galaxy counts given in Figure 1 of McCracken et al. 2000 (MNRAS 311,707):


According to Figure 1, a density of 2.9 \(10^4\) galaxies per square degree is reached at \(K=19.7\) \((K_{AB}=21.6)\) — in excellent agreement with Stefano’s value of \(K_{AB}=21.5\) for the CADIS counts.

The stellar curve in Figure 1 was constructed by extrapolating and integrating the differential counts given in Figure 2 of the 2Mass web documentation:

http://www.ipac.caltech.edu/2mass/releases/second/

Since the latter counts are only complete down to \(K=14.5\), the amount of extrapolation to the magnitudes of interest at \(K>19\) is considerable — especially since one does expect to run out of stars at some point at high latitudes. Fortunately, since the total counts at the faint magnitudes of interest are dominated by galaxies, this uncertainty does not appreciably affect the conclusions. Nonetheless, the stellar curve in Figure 1 predicts an integrated density of 5100 stars per square degree at \(K<19.3\) \((K_{AB}<21.2)\) — which is somewhat higher than the density of \(~3000\) per square degree at this magnitude quoted by Stefano, but still a mere \(~20\%\) of the total counts at this magnitude.

I conclude that there appears to be very good agreement as to the expected source counts in the high galactic latitude case.
b) Miscellaneous Notes

i) The slope of the high latitude counts at the magnitude range of interest is \( \frac{\text{d} \log N(<K)}{\text{d} m} = \alpha \sim 0.35 \). Hence I again agree with Stefano that the inferred contrast scales almost linearly with the “critical density” or “spoiler fraction” that one is willing to accept (actually to the power of \( 1/2.5 \alpha \sim 1.1 \)).

ii) When going down in Galactic latitude the stellar component obviously increases in importance. Roughly speaking, at longitudes away from the Galactic center a given object density is reached at \( \sim 1 \text{ mag brighter at } \text{b}=30^\circ \), compared to the \( \text{b}=90^\circ \) case. Comparing Figure 1 of Kümmel & Wagner (2000) A&A 353, 867

http://link.springer.de/link/service/journals/00230/papers/0353003/2300867/small.htm
to the 2MASS curve used above seems to shows this nicely.

Hence I again agree with Stefano that increasing the inferred contrast at the poles by a factor of \( \sim 2 \) or so should take care of things down to \( \text{b}=30^\circ \).

c) Low Latitude Case

The more thorny issue is clearly how to deal with the low Galactic latitudes where most of the NGST Galactic science takes place. Although a more careful appraisal involving realistic stellar fields in clusters and star forming regions is clearly called for, as an example Figure 2 gives the integrated star counts in the Galactic plane obtained by numerically integrating the differential counts of Figure 3 (which refers to Galactic longitude \( l=55^\circ \)) of the 2Mass web documentation (URL given above). Since the stellar densities of interest are reached at relatively bright magnitudes, the required extrapolation in this case is minimal.

![Figure 2 – Integrated counts in the Galactic plane (l=55°)](image-url)
2) The Critical Density

The only disagreement I have with Stefano’s analysis lies in his adopted value of the “critical density” which was taken from Jeremy’s note. In particular, I believe Jeremy’s value of 2.9 \(10^4\) galaxies per square degree is on the low side.

The key issue here is obviously how large a solid angle a dispersed spectrum extends on the sky (Figure 3).

![Potential contamination zone around target](image)

Figure 3 - The problem of overlapping spectra

Both current ESA MEMS NIRSPEC designs assume that the 1-5 µm spectra region will be covered at R=1000 in three separate grating settings and sampled at a plate scale of 100 mas per (square) detector pixel. Since grating spectrographs have dispersion curves with nearly constant \(\Delta\lambda\) per pixel, the length of the dispersed image spanning a given range at a given resolution can easily be calculated. Table 1 gives the estimated spectral lengths for three realistic bandpasses (corresponding to three first order gratings), assuming R=1000 is set in the middle of the band and two-pixel sampling.

<table>
<thead>
<tr>
<th>Grating Range (µm)</th>
<th>Length (pixels)</th>
<th>Width (mas)</th>
<th>Adopted Width (pixels)</th>
<th>Contamination Zone (square degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0 - 1.8</td>
<td>1142</td>
<td>84 - 151</td>
<td>2</td>
<td>7.0 (10^{-6})</td>
</tr>
<tr>
<td>1.7 - 3.0</td>
<td>1106</td>
<td>143 - 252</td>
<td>3</td>
<td>1.0 (10^{-5})</td>
</tr>
<tr>
<td>2.9 - 5.0</td>
<td>1063</td>
<td>243 - 419</td>
<td>4</td>
<td>1.3 (10^{-5})</td>
</tr>
</tbody>
</table>

As anticipated, one needs of order N~R=10^3 pixels to cover roughly one octave of spectrum at R=10^3.

The second issue is what width to adopt for the spectra in the spatial direction. Since each spectrum spans almost an octave in wavelength, some flaring of the PSF is expected along the dispersion direction due to diffraction. Table 1 gives the expected span in width of the image in each band, calculated for a D=6 m aperture from the expression \(w=2.44(\lambda/D)\) giving the full width out to the first dark ring of an (unobstructed) Airy pattern. Also listed is the adopted “rounded” width of the spectrum in (100 mas) pixels.

Ignoring the complication of the flaring (which, as pointed out by Stefano, is masked by the finite extent of the target galaxies in any case) the solid angle extended by each spectrum is easily calculated from the entries in Table 1. Since both the target image and the contaminating leaking “spoiler” image have the same extent on the sky, the size of the “Contamination Zone” around a given target in which overlap can occur is four times the solid angle extended by a single spectrum (Figure 3). Aside from the slightly shorter spectra, the introduction of this factor of 4 is the main difference w.r.t. Jeremy’s deliberations.
3) Is there an argument here?

As far as the high latitude case is concerned, no – we’re merely propagating slight differences in assumed input parameters.

If we stick to the middle 1.7 - 3.0 µm band in Table 1 (for which the K-band counts are directly applicable) I get that the 10% contamination critical density is $1.0 \times 10^4$ objects per square degree, which is 2.9 times lower than Jeremy’s value as adopted by Stefano.

All other factor equal, this should lead to a difference in critical magnitude given by $\Delta m = \Delta \log N/\alpha = \log(2.9)/0.35 = 1.3$

Indeed, according to Figure 1, $1.0 \times 10^4$ objects per square degree is reached at a critical magnitude of $K=18.2$ ($K_{AB}=20.1$), compared to Stefano’s value of $K_{AB}=21.5$. It follows that our difference in assumed source counts amounts to no more than 0.1 mag, or ~10%.

Another difference lies in our assumed limiting magnitude, which I in my previous email took to be $K_{AB}=27$ as stated in DRM 7 (now 15), but Stefano assumes to be $K_{AB}=26.6$. On this point I am inclined to adopt Stefano’s value since it takes into account the $\Delta m \approx 0.4$ mag across-the-board loss in limiting sensitivity incurred by the telescope having shrunk from 8 m to 6 m since the relevant DRM was written (gee, have I really been in denial about that one?).

Thus my current “best” estimate of the high latitude contrast requirement corresponding to the 10%/10% pain criterion still comes out to: $26.6 - 20.1 + 2.5 +1.0 = 10$ mags = $10^4$ – where I’ve thrown in the extra magnitude to go down to $b=30^\circ$.

The factor ~5 difference w.r.t. Stefano’s value of 1800 - 2300 is fully explained by a factor ~3.4 difference due to my adopting a 2.9 times larger “contamination zone”, combined with another factor ~1.5 difference due to my adding a full magnitude of attenuation (factor ~2.5) to go to $b=30^\circ$, compared to Stefano’s smaller factor 1.6 (~0.5 mag) increase.

4) Conclusion?

Stefano and I are clearly in violent agreement as to the formulation of the basic problem and what the sky ought to look like at high Galactic latitudes – and have merely adopted somewhat different input parameters.

In other words, we are clearly dealing with a continuous curve, combined with a judgement call as to where the proper “pain threshold” should be set. While we can argue about factors of two or three until we are blue in the face, it would seem indisputable that the extragalactic DRM science case requires a minimum contrast somewhere between $10^3$ and $10^4$.

The key issue is clearly the low latitude “Galactic science” case. Naively applying the same logic to Figure 2 leads to a whopping contrast of $26.6 - 13.0 + 2.5 = 16.1$ mag = $2.8 \times 10^6$ – a number that presumably can only be reached with a real physical aperture plate.