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Structure Detection in Low Intensity X-Ray Images using the Wavelet Transform Applied to Galaxy Cluster Cores Analysis

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Abstract. In the context of assessing and characterizing structures in X-ray images, we compare different approaches. The intensity level is often very low and necessitates a special treatment of Poisson statistics. The method based on wavelet function histogram is shown to be the most reliable one. Multi-resolution filtering methods based on the wavelet coefficients detection are also discussed. Finally, using a set of ROSAT HRI deep pointings, the presence of small-scale structures in the central regions of clusters of galaxies is investigated.

1. Wavelet Coefficient Detection

The ability to detect structures in X-ray images of celestial objects is crucial, but the task is highly complicated due to the low photon flux, typically from 0.1 to a few photons per pixel. Point sources detection can be done by fitting the Point Spread Function, but this method does not allow extended sources detection. One way of detecting extended features in a image is to convolve it by a Gaussian. This increases the signal to noise ratio, but at the same time, the resolution is degraded. The VTP method (Scharf et al. 1997) allows detection of extended objects, but it is not adapted to the detection of substructures. Furthermore, in some cases, an extended object can be detected as a set of point sources (Scharf et al. 1997). The wavelet transform (WT) has been introduced (Slezak et al. 1990) and presents considerable advantages compared to traditional methods. The key point is that the wavelet transform is able to discriminate structures as a function of scale, and thus is well suited to detect small scale structures embedded within larger scale features. Hence, WT has been used for clusters and subclusters analysis (Slezak et al. 1994; Grebenev et al. 1995; Rosati et al. 1995; Biviano et al. 1996), and has also allowed the discovery of a long, linear filamentary feature extending over approximately 1 Mpc from the Coma cluster toward NGC 4911 (Vikhlinin et al. 1996). In the first analyses of images by the wavelet transform, the Mexican hat was used. More recently the *à trous* wavelet transform algorithm has been used because it allows an easy reconstruction (Slezak et al. 1994; Vikhlinin et al. 1996). By this algorithm, an image $I(x, y)$ can be decomposed into a set $(w_1, ..., w_n, c_n)$,

$$I(x, y) = c_n(x, y) + \sum_{j=1}^{n} w_j(x, y) \qquad (1)$$

Several statistical models have been used in order to say whether an X-ray wavelet coefficient $w_j(x,y)$ is significant, i.e., not due to the noise. In Viklinin et al. (1996), the detection level at a given scale is obtained by an hypothesis that the local noise is Gaussian. In Slezak et al. (1994), the Anscombe transform was used to transform an image with Poisson noise into an image with Gaussian noise. Other approaches have also been proposed using k sigma clipping on the wavelet scales (Bijaoui & Giudicelli 1991), simulations (Slezak et al. 1990, Escalera & Mazure 1992, Grebenev et al. 1995), a background estimation (Damiani et al. 1996; Freeman et al. 1996), or the histogram of the wavelet function (Slezak et al. 1993; Bury 1995). Simulations have shown (Starck and Pierre, 1997) that the best filtering approach for images containing Poisson noise with few events is the method based on histogram autoconvolutions. This method allows one to give a probability that a wavelet coefficient is due to noise. No background model is needed, and simulations with different background levels have shown the reliability and the robustness of the method. Other noise models in the wavelet space lead to the problem of the significance of the wavelet coefficient.

This approach consists of considering that, if a wavelet coefficient $w_j(x,y)$ is due to the noise, it can be considered as a realization of the sum $\sum_{k \in K} n_k$ of independent random variables with the same distribution as that of the wavelet function (n_k being the number of photons or events used for the calculation of $w_j(x,y)$). Then we compare the wavelet coefficients of the data to the values which can taken by the sum of n independent variables. The distribution of one event in the wavelet space is directly given by the histogram H_1 of the wavelet ψ. Since independent events are considered, the distribution of the random variable W_n (to be associated with a wavelet coefficient) related to n events is given by n autoconvolutions of H_1: $H_n = H_1 \otimes H_1 \otimes ... \otimes H_1$

For a large number of events, H_n converges to a Gaussian. Knowing the distribution function of $w_j(x,y)$, a detection level can be easily computed in order to define (with a given confidence) whether the wavelet coefficient is significant or not (i.e not due to the noise).

Significant wavelet coefficients can be grouped into structures (a structure is defined as a set of connected wavelet coefficients at a given scale), and each structure can be analyzed independently. Interesting information which can be easily extracted from an individual structure includes the first and second order moments, the angle, the perimeter, the surface, and the deviation of shape from sphericity (i.e., $4\pi \frac{Surface}{Perimeter^2}$). From a given scale, it is also interesting to count the number structures, and the mean deviation of shape from sphericity.

2. Image Filtering

In the previous section, we have shown how to detect significant structures in the wavelet scales. A simple filtering can be achieved by thresholding the non-significant wavelet coefficients, and by reconstructing the filtered image by the inverse wavelet transform. In the case of the *à trous* wavelet transform algorithm, the reconstruction is obtained by a simple addition of the wavelet scales and the last smoothed array. The solution S is:
$S(x,y) = c_p^{(I)}(x,y) + \sum_{j=1}^{p} M(j,x,y) w_j^{(I)}(x,y)$

where $w_j^{(I)}$ are the wavelet coefficients of the input data, and M is the multiresolution support ($M(j,x,y) = 1$, the wavelet coefficient at scale j and at position (x,y) is significant). A simple thresholding generally provides poor results. Artifacts appear around the structures, and the flux is not preserved. The multiresolution support filtering (see Starck et al (1995)) requires only a few iterations, and preserves the flux. The use of the adjoint wavelet transform operator (Bijaoui et Rué, 1995) instead of the simple coaddition of the wavelet scale for the reconstruction suppresses the artifacts which may appear around objects. Partial restoration can also be considered. Indeed, we may want to restore an image which is background free, objects which appears between two given scales, or one object in particular. Then, the restoration must be performed without the last smoothed array for a background free restoration, and only from a subset of the wavelet coefficients for the restoration of a set of objects (Bijaoui et Rué 1995).

3. Galaxy Cluster Cores Analysis

Cluster cores are thought to be the place where virialisation first occurs and thus in this respect, should present an overall smooth distribution of the X-ray emitting gas. However, in cooling flows (CF) - and most probably in the whole ICM - the presence of small scale inhomogeneities is expected as a result of the development of thermal instability (e.g., Nulsen 1986). (Peculiar emission from individual galaxies may be also observed, although at the redshifts of interest in the present paper (≥ 0.04) - and S/N - such a positive detection would be most certainly due to an AGN.) It is thus of prime interest to statistically investigate at the finest possible resolution, the very center of a representative sample of clusters, in terms of luminosity, redshift and strength of the cooling flow.

Using a set of ROSAT HRI deep pointings, the shape of cluster cores, their relation to the rest of the cluster and the presence of small scale structures have been investigated (Pierre & Starck, 1997). The sample comprises 23 objects up to z=0.32, 13 of them known to host a cooling flow. Structures are detected and characterized using the wavelet analysis described in section 1.

We can summarize our findings in the following way:

- In terms of shape of the smallest central scale, we find no significant difference between, CF and non CF clusters, low and high z clusters.
- In terms of isophote orientation and centroid shift, two distinct regions appear and seem to co-exist: the central inner 50-100 kpc and the rest of the cluster. We find a clear trend for less relaxation with increasing z.
- In general, very few isolated “filaments” or clumps are detected above 3.7σ in the cluster central region out to a radius of ~ 200 kpc. Peculiar central features have been found in a few high z clusters.

This study, down to the limiting instrumental resolution, enables us to isolate - in terms of dynamical and physical state - central regions down to a scale comparable to that of the cluster dominant galaxy. However it was not possible to infer firm connections between central morphologies and cooling flow rates or redshift. Our results allow us to witness for the first time at the cluster center, the competition with the relaxation processes which should here be well advanced and local phenomena due to the presence of the cD galaxy. Forthcoming AXAF and XMM observations at much higher sensitivity, over a wider spectral range and with a better spatial resolution may considerably improve our understanding of the multi-phase plasma and of its inter-connections with the interstellar medium.

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